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DETERMINATION OF THE VELOCITY OF THE PHASE DRIFT IN A VERTICAL BUBBLE FLOW

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The velocity of the phase drift in a vertical bubble flow is calculated approximately on the basis of a method developed earlier.

In [1] the process of gas-bubble floating in a vertical tube filled with an ideal fluid was studied. On the basis of an approach developed by D. A. Labuntsov the sought flow was constructed by the method of superposition of (a) a flow from a point mass source with an asymptotic velocity u_0 for $z \to \pm\infty$ and (b) a uniform flow with a velocity u_∞ directed opposite to the z axis (Fig. 1). As a result a pattern of a tube flow of an ideal fluid past an axisymmetric solid body with a velocity $w_\infty = u_\infty - u_0$ for $z \to \infty$ and a velocity $u_\infty + u_0$ in the cylindrical gap between the tube wall and the body was obtained. Passage to the case of a flow past a bubble was made in [1] by assigning "the condition of a free surface" [2] in the vicinity of the front critical point K of the streamlined body (Fig. 2). As a result the dependence of the sought characteristic of the flow (the Froude number $\mathrm{F} = w_\infty/\sqrt{gR_0}$) on the dimensionless distance $\hat{z}_0$ between the source coordinate O and the front critical point K is obtained (Fig. 3). Since in a real case of fluid flow past a floating bubble some unique value of the Froude number should be realized, to close the problem an additional condition, namely, the "free-surface condition" at the point of the source O, was used in [1]. This gives the following pair of values: $z_0 = z_{0*} \approx 0.58$; $\mathrm{F} = \mathrm{F}_* \approx 0.486$. In [3] passage to the limit as $z \to \infty$ was performed to close the problem, thus giving $\mathrm{F} = \mathrm{F}_\infty \approx 0.511$. The values of the Froude number obtained in [1, 3] are in a good agreement with experimental data of [4] on the velocity of floating of gas bubbles in a vertical tube.

As is known, a model of the phase drift is widely used to determine the hydrodynamic characteristics of bubble flows [5]. It includes various empirical relations relating the velocity of bubble motion relative to the fluid flow (the drift velocity) to the volumetric void fraction of the bubble flow φ. The method developed in [1, 3] makes it possible to calculate drift velocities within the framework of an approximate physical model of the flow in an elementary cell of a two-phase flow. Figure 2 presents a pattern of inflow of an ideal fluid with a velocity $w_\infty = u_\infty - u_0$ onto a stationary semi-infinite bubble with the following asymptotic value of the radius of the cylindrical side surface:

$$z \to -\infty, \quad b = \frac{R_1}{R_0} = \left(\frac{2p}{1+p}\right)^{1/2}, \tag{1}$$

where $p = u_0/u_\infty$ is the ratio of the velocities of the source and the uniform flow.

The velocity of the flow in the cylindrical gap between the bubble and the wall is $u_\infty + u_0$. According to the available experimental data [4], the velocity of bubble floating does not depend on the gap length. Therefore, the scheme in Fig. 2 can be replaced approximately by the pattern of a fluid flow past a bubble of radius R_1 with a plane rear surface (Fig. 4a). Then, within the considered approximation of an ideal fluid the tube wall can be replaced by the cylindrical side surface of some elementary fluid cell at the center of which the gas bubble is located. Passage to the actual problem of bubble floating in a stationary fluid is performed by superposition of the velocity $w_\infty = u_\infty - u_0$ on the flow considered. Then, the velocity of the flow in the side gap of the cell is $2u_0$ (Fig. 4b).

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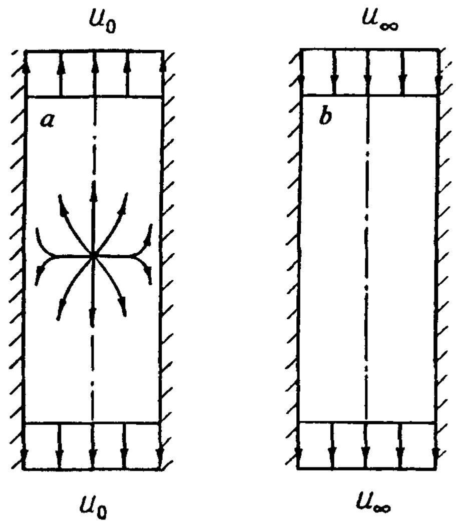

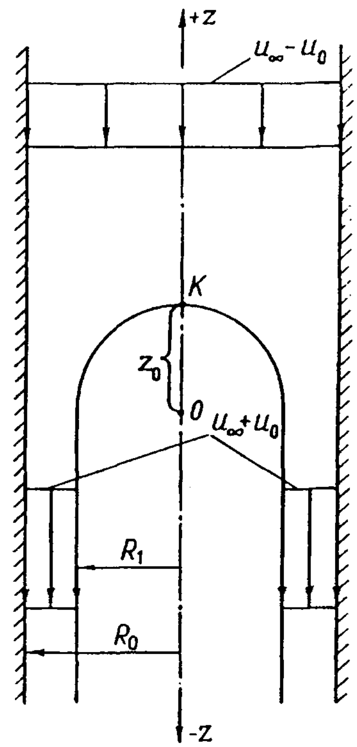

Fig. 1. Construction of an ideal-fluid flow in a tube by the method of superposition: a) point mass source; b) uniform flow.

Fig. 2. Pattern of a flow past a gas bubble.

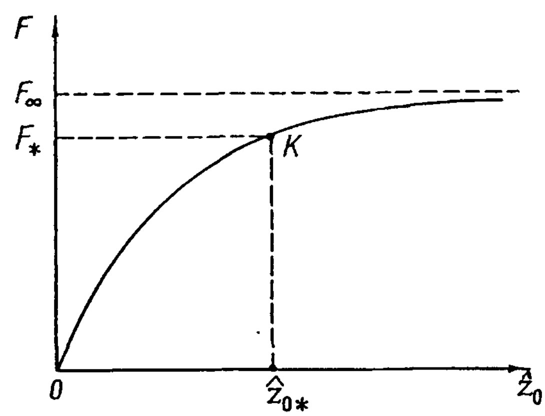

Fig. 3. Dependence of the Froude number on the parameter $\hat{z}_0$.

The generally adopted interpretation of the existence of phase drift is reduced to the qualitative assertion of the presence of interaction between neighboring cells that leads to an ascending flow (drift) of the cell and increases with the volumetric void fraction [4, 5]. Within the framework of the present model we determine the velocity of the phase drift by adding a certain kinematic condition to the obtained pattern of the flow in the cell, viz., we superimpose on the flow (Fig. 4b) an additional ascending flow (drift) for which two relative velocities of fluid flow in the cell (in front of the bubble and in the side gap) will be equal (Fig. 4c). The mentioned condition of the drift leads to the following relations: a) the relative velocities of fluid flow in the cell are equal to the asymptotic velocity of the source u_0; b) the absolute velocity of bubble floating relative to the volume of fluid stationary at infinity is equal to the velocity of the uniform flow u_∞. We introduce the dimensionless drift factor

$$D \equiv \frac{u_\infty}{\omega_\infty} = \frac{u_\infty}{u_\infty - u_0}. \quad (2)$$

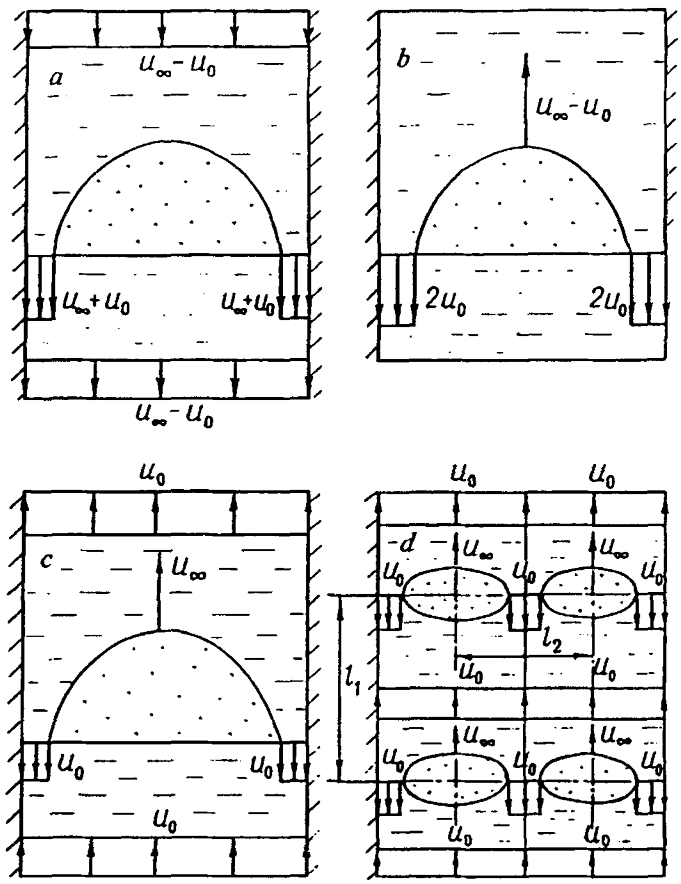

Fig. 4. Scheme of flow in a cell: a) fluid inflow onto a stationary bubble; b) bubble floating in a stationary cell; c) bubble floating in a cell with a superimposed drift velocity; d) floating of a "grid" of bubbles.

The quantity D is related to the parameter b of (1) by the relation

$$D = \frac{2 - b^2}{2\,(1 - b^2)}\,. \tag{3}$$

For $b \to 0$, $D \to 1$ we obtain the limiting case of floating of a single bubble in an infinite volume of fluid [3]: the bubble dimensions are vanishingly small compared to the cell radius, and the drift velocity of the cell is equal to zero. The case $b \to 1$, $D \to \infty$ corresponds to joining of the side surfaces of neighboring bubbles: the velocity of fluid flow in a side gap of vanishingly small thickness increases without limit; the velocity of cell drift tends to infinity.

To obtain the resultant dependence of the drift factor D on the volumetric void fraction of the two-phase flow φ it is necessary to relate the latter quantity to the parameter b. We consider the following case of floating of a "grid" of bubbles: a) all the bubbles have the shape of oblate spheroids with respect to the vertical axis with a ratio of the semiaxes of 4:1 [2]; b) the distances between the closest points of the surface of neighboring bubbles along the horizontal and the vertical are equal to each other: $l_1 \approx l_2$ (Fig. 4d). As a result we obtain the sought relation in parametric form:

$$D = 1 + \frac{1}{2\,(1 - b)\,(3 - b)}\,, \tag{4}$$

$$\varphi = \frac{1}{(2 - b)^2\,(5 + 4b)}\,. \tag{5}$$

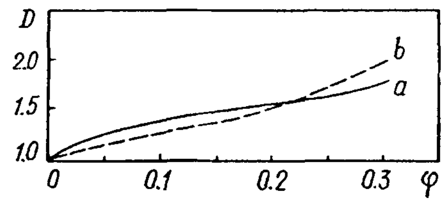

Fig. 5. Comparison of results of drift-velocity calculations by (4), (5) of the present work (a) and the empirical formula of [4, 5] (b).

At present the empirical relation [4, 5]

$$D = \frac{1}{(1 - \varphi)^{1.75}} \tag{6}$$

is commonly adopted for the range of volumetric void fractions $0 < \varphi \leq 0.3$ (bubble flow). As is seen from Fig. 5, relations (4), (5) of the present work are in satisfactory agreement with the empirical formula (6) within the entire range of its applicability.

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NOTATION

z, axial coordinate; R_0, tube radius; R_1, radius of the cylindrical side surface of the body for $z \to -\infty$; u_0, asymptotic velocity of the point mass source; u_∞, velocity of uniform flow; $w_\infty = u_\infty - u_0$, velocity of fluid inflow onto the bubble for $z \to \infty$; g, acceleration of gravity; $F = w_\infty/\sqrt{gR_0}$, Froude number; z_0, dimensionless distance between the source coordinate and the front critical point of the streamlined body; $b = R_1/R_0$, ratio of the body radius to the tube radius; $D = u_\infty/w_\infty$, drift factor; φ, volumetric void fraction of the bubble flow.

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